

UGANDA ADVANCED CERTIFICATE OF EDUCATION

SET 9

Paper 1

3 hours

INSTRUCTIONS TO CANDIDATES:

- Attempt **ALL** the questions in section **A** and only **FIVE (5)** from section **B**.
- All working must be clearly shown.
- Mathematical tables with list of formulae and squared paper are provided.
- Clearly indicate the questions you have attempted in section B in a grid on your answer scripts. **DONOT** hand in question paper.

Qn									
Marks									

SECTION A (40 MARKS)

1. Solve the simultaneous equations:
 $3y + x - 3z = -4$, $3x - y + 2z = 1$, $-2x + y + z = 7$ (5 marks)
2. Prove that $\tan^2\left(\frac{\pi}{4} + \theta\right) = \left(\frac{1 + \sin 2\theta}{1 - \sin 2\theta}\right)$. (5 marks)
3. The first term of an Arithmetic progression and Geometric progression are each $\frac{2}{3}$. Their common difference and common ratio are each to x and the sum of their first three terms are also equal. Find the two possible values of x . (5 marks)
4. Find the point of intersection between the lines $\mathbf{r} = (-2\mathbf{i} + 5\mathbf{j} - 1\mathbf{k}) + \lambda(3\mathbf{i} + \mathbf{j} + 3\mathbf{k})$, and $\mathbf{r} = (8\mathbf{i} + 9\mathbf{j}) + t(4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$. (5 marks)
5. Evaluate $\int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{2}} \right) dx$, giving your answer in simplified surd. (5 marks)
6. $A(-3, 0)$ and $B(3, 0)$ are fixed points. Show that the locus of a point $P(x, y)$ which moves such that $PB = 2PA$ is a circle and find its centre and radius. (5 marks)
7. Air is pumped into a spherical balloon at a rate of $256\pi \text{ cm}^3 \text{ s}^{-1}$. When the radius of the balloon is 15 cm , find the rate at which its radius is increasing. (5 marks)
8. Find the equation of the tangent to the curve $x^2y - xy^2 = 12$ at the point where $(4, 3)$. (5 marks)

SECTION B

- 9a) Solve the equation: $\sqrt{(y+6)} - \sqrt{(y+3)} = \sqrt{(2y+5)}$. Check your answers. (6 marks)
- b) Solve for x and y : $\begin{matrix} \log(x+y) = 1 \\ 2\log y - \log(30-x) = 0 \end{matrix}$. (6 marks)
- 10a) Show that $1 + 2i$ is a root of the equation $2z^3 - z^2 + 4z + 15 = 0$, hence find the other roots. (6 marks)

- b) If $z = 1 + 2i$ is a root of the equation $z^3 + az + b = 0$ where a and b are real, find the values of a and b . (6 marks)
- 11a) Differentiate from first principles $y = \frac{1}{\sqrt{x}}$. (5 marks)
- b) A hemispherical bowl is being filled with water at a uniform rate. When the height of the water is h cm the volume is $\pi(rh^2 - \frac{1}{3}h^3)$ cm³, r cm being the radius of the hemisphere. Find the rate at which the water level is rising when it is half way to the top, given that $r = 6$ cm and the bowl fills in 1 min. (7 marks)
- 12a) Prove that $\tan \frac{1}{2}(A - B) + \tan \frac{1}{2}(A + B) = \frac{2 \sin A}{\cos A + \cos B}$. (5 marks)
- b) If $\tan \alpha = p$, $\tan \beta = q$, $\tan \gamma = r$, prove that $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$, hence, show that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{\pi}{4}$. (7 marks)
- 13a) The profit y generated from the sale is given by the function $y = 72x + 3x^2 - 2x^3$. Calculate how many terms should be sold to receive maximum profit and determine the maximum profit. (5 marks)
- b) Find the area enclosed by the curve $y = x(8 - x)$ and the line $y = 12$. (7 marks)
14. Determine the turning points and asymptotes of the curve $y = \frac{4x^2 - 10x + 7}{(x - 1)(x - 2)}$ hence, sketch the curve. (12 marks)
- 15a) If the position vectors of points A and B are $2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ respectively, find the position vector of the point P which divides $\mathbf{AB}$ externally in the ratio 5:3. (6 marks)
- b) Find the Cartesian equation of a plane through the origin parallel to the lines $\mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 4 \\ -5 \\ -8 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 7 \\ -6 \end{pmatrix}$. (6 marks)

16. Given that the line $y = mx + c$ is a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, show that $c^2 = a^2m^2 + b^2$. Hence, determine the equations of the tangents at the point $(-3, 3)$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. (12 marks)

END

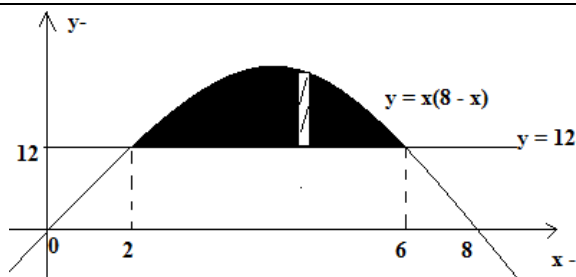
SET 9 MARKING GUIDE 2025

1.	$x + 3y - 3z = -4$, (i) $3x - y + 2z = 1$, (ii) $-2x + y + z = 7$. (iii) $3 \text{ eqn (i)} - \text{eqn (ii)}, 10y - 11z = -13 \dots \text{(iv)}$ $2 \text{ eqn (i)} + \text{eqn (iii)}, 7y - 5z = -1 \dots \text{(v)}$ $7 \text{ eqn (iv)} - 10 \text{eqn (v)}, -27z = -81, z = 3, y = 2, x = -1$.	
2	$\tan^2\left(\frac{\pi}{4} + \theta\right) = \left(\frac{\tan \frac{\pi}{4} + \tan \theta}{1 - \tan \frac{\pi}{4} \tan \theta}\right)^2 = \left(\frac{1 + \tan \theta}{1 - \tan \theta}\right)^2 = \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}\right)^2$ $= \left(\frac{\cos^2 \theta + 2 \cos \theta \sin \theta + \sin^2 \theta}{\cos^2 \theta - 2 \sin \theta \cos \theta + \sin^2 \theta}\right) = \left(\frac{1 + \sin 2\theta}{1 - \sin 2\theta}\right)$	
3	$a + a + x + a + 2x = a + ax + ax^2$ $3 \cdot \frac{2}{3} + 3x = \frac{2}{3}(1 + x + x^2)$ $2x^2 - 7x - 4 = 0, \Leftrightarrow (x - 4)(2x + 1) = 0$ $x = 4, x = -\frac{1}{2}$	
4	$\begin{pmatrix} -2 \\ 5 \\ -11 \end{pmatrix} + \begin{pmatrix} 3\lambda \\ \lambda \\ 3\lambda \end{pmatrix} = \begin{pmatrix} 8 \\ 9 \\ 0 \end{pmatrix} + \begin{pmatrix} 4t \\ 2t \\ 5t \end{pmatrix}$ $\Rightarrow -2 + 3\lambda = 8 + 4t \dots \text{(i)}$ $5 + \lambda = 9 + 2t \dots \text{(ii)}$ $-11 + 3\lambda = 5t \dots \text{(iii)}$ $\begin{array}{rcl} \text{Eqn(i)} - \text{eqn(ii)} \times 3 & -2 + 3\lambda & = 8 + 4t \\ & -(15 + 3\lambda) & = -(27 + 6t) \end{array} \text{ to get}$ $-17 = -19 - 2t$ $\Rightarrow t = -1 \text{ then from eqn(i)} \lambda = 2$ $\text{Substitute } t \text{ \& } \lambda \text{ in (i), LHS. } -11 + 6 = -5 = \text{RHS}$ $= 4\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}$	

5	$\int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{2}} \right) dx = \int_1^4 \left(x^{-1/2} + \frac{1}{\sqrt{2}} \right) dx$ $= \left[2x^{1/2} + \frac{x}{\sqrt{2}} \right]_1^4$ $= \left(2\left(\sqrt{4} + \frac{4}{\sqrt{2}} \right) \right) - \left(2\sqrt{1} + \frac{1}{\sqrt{2}} \right)$ $= 4 + 2\sqrt{2} - \left(\frac{4 + \sqrt{2}}{2} \right)$ $= 2 + \frac{3}{2}\sqrt{2}$	
6	$\sqrt{(x-3)^2 + (y-0)^2} = 2\sqrt{(x+3)^2 + (y-0)^2}$ $x^2 - 6x + 9 + y^2 = 4(x^2 + 6x + 9 + y^2)$ $3x^2 + 3y^2 + 30x + 27 = 0$ $x^2 + y^2 + 10x + 9 = 0, \quad (x+5)^2 + (y-0)^2 = 16$ Which is an equation of a circle with centre $(-5, 0)$ and $r = 4$	
7	Vol of a sphere is $V = \frac{4}{3}\pi r^3$ and the rate $\frac{dV}{dt} = 256\pi \text{ cm}^3 \text{ s}^{-1}$ Vol of hemisphere is $\frac{dV}{dr} = 4\pi r^2$, so $\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dr}{dt} = \frac{1}{4\pi r^2} \times 256\pi \text{ for } r = 16, \quad \frac{dr}{dt} = \frac{1}{4\pi(16)^2} \times 256\pi = \frac{64}{225} \text{ cm s}^{-1}$	
8	$2xy + x^2 \frac{dy}{dx} - y^2 - 2xy \frac{dy}{dx} = 0, \quad \frac{dy}{dx} = \frac{y^2 - 2xy}{x^2 - 2xy}$ $(4, 3), \quad \frac{dy}{dx} = \frac{9 - 24}{16 - 24} = \frac{-15}{-8} = \frac{15}{8}$ Equation: $\frac{y-3}{x-4} = \frac{15}{8}, \quad 8y = 15x - 36$	
9.	$\sqrt{y+6} - \sqrt{y+3} = \sqrt{2y+5}, \text{ square both sides}$ $y+6 + y+3 - 2\sqrt{(y+6)(y+3)} = 2y+5, \quad \sqrt{y^2+9y+18} = 2$ $y^2+9y+18=4, \quad y^2+9y+14=0, \quad (y+7)(y+2)=0,$ $y = -7, \quad y = -2$ Check: $\sqrt{(y+6)} - \sqrt{(y+3)} = \sqrt{(2y+5)}$ $y = -7, \text{ L.H.S} = \sqrt{-1} - \sqrt{-4} \neq \sqrt{-9} \neq \text{R.H.S}$ $x = -2, \text{ L.H.S} = \sqrt{4} - \sqrt{1} = \sqrt{1} = \text{R.H.S}$ Therefore, $x = -2$, is the only correct solution.	
b)	$x + y = 10 \dots\dots\dots(i) \quad y = 10 - x$	

	$\frac{y^2}{30-x} = 1, \quad y^2 = 30 - x \dots\dots(ii)$ $(10-x)^2 = 30 - x$ $x^2 - 19x + 70 = 0, \quad (x-5)(x-14) = 0$ $x = 5, \quad x = 14 \text{ corresponding values are } y = 5, \quad y = -4 \text{ respectively.}$	
10 a)	If $z = 1 + 2i$ is a root, then $\bar{z} = 1 - 2i$ the conjugate root is the other root. Sum of roots is $1 + 2i + 1 - 2i = 2$ and product is $(1 + 2i)(1 - 2i) = 5$ thus the equation is $z^2 - 2z + 5 = 0$. $z^2 - 2z + 5 \overline{) 2z^3 - z^2 + 4z + 15}$ By long division $\begin{array}{r} 2z^3 - 4z^2 + 10z \\ \underline{3z^2 - 6z + 15} \\ 3z^2 - 6z + 15 \end{array}$ Thus $2z + 3 = 0$ gives us $z = -\frac{3}{2}$ Other roots are $2 + i, -\frac{3}{2}$	
b)	$(1 + 2i)^3 + a(1 + 2i) + b = 0$ $-11 - 2i + a + 2ai + b = 0, \quad (-11 + a + b) + (2a - 2)i = 0$ Thus, $2a - 2 = 0, \quad a = 1$ $(-11 + 1 + b) = 0, \quad b = 10$	
11 a)	$y + \partial y = \frac{1}{\sqrt{x + \partial x}}, \quad \partial y = \left(\frac{1}{\sqrt{x + \partial x}} \right) - \frac{1}{\sqrt{x}}$ $\partial y = \frac{\sqrt{x} - \sqrt{x + \partial x}}{\sqrt{x}(\sqrt{x + \partial x})}, \quad \partial y = \frac{\sqrt{x} - \sqrt{x + \partial x}}{\sqrt{x}(\sqrt{x + \partial x})} \times \frac{(\sqrt{x} + \sqrt{x + \partial x})}{(\sqrt{x} + \sqrt{x + \partial x})}$ $\partial y = \frac{x - x - \partial x}{\sqrt{x}(\sqrt{x + \partial x})(\sqrt{x} + \sqrt{x + \partial x})}, \quad \frac{\partial y}{\partial x} = \frac{-1}{\sqrt{x}(\sqrt{x + \partial x})(\sqrt{x} + \sqrt{x + \partial x})}$ As $\partial x \rightarrow 0, \quad \frac{\partial y}{\partial x} \rightarrow \frac{dy}{dx}, \text{ so } \frac{dy}{dx} = \frac{-1}{2x^{3/2}}$	
b)	Vol of hemisphere is $V = \frac{2}{3}\pi r^3$ and the rate $\frac{dV}{dt} = \frac{\pi r^3}{90} \text{ cm}^3 \text{ s}^{-1}$ $V = \pi \left(rh^2 - \frac{1}{3}h^3 \right), \quad \frac{dV}{dh} = \pi(2rh - h^2)$ So, $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}, \quad \frac{dh}{dt} = \frac{1}{\pi(2rh - h^2)} \times \frac{\pi r^3}{90} \text{ for } r = 6, \quad h = 3$ $\frac{dh}{dt} = \frac{216}{90 \times 27} = \frac{4}{45} \text{ cm s}^{-1}$	

12 a)	From the L.H.S $\tan \frac{1}{2}(A - B) + \tan \frac{1}{2}(A + B) = \frac{\sin \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A - B)} + \frac{\sin \frac{1}{2}(A + B)}{\cos \frac{1}{2}(A + B)}$ $= \frac{\cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B) + \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)}{\cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)}$ $= \frac{\frac{1}{2}(\sin A - \sin B) + \frac{1}{2}(\sin A + \sin B)}{\frac{1}{2}(\cos A + \cos B)}$ $= \frac{2 \sin A}{\cos A + \cos B} \text{ as the R.H.S}$	
b)	From the L.H.S, $\tan(\alpha + \beta + \gamma) = \tan((\alpha + \beta) + \gamma)$ $= \frac{\tan(\alpha + \beta) + \tan \gamma}{1 - \tan(\alpha + \beta) \tan \gamma}$ $= \frac{\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} + \tan \gamma}{1 - \left[\frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \right] \tan \gamma}$ $= \frac{\tan \alpha + \tan \beta + \tan \gamma - \tan \alpha \tan \beta \tan \gamma}{1 - \tan \alpha \tan \gamma - \tan \alpha \tan \beta - \tan \beta \tan \gamma}$ $\tan(\alpha + \beta + \gamma) = \frac{p + q + r - pqr}{1 - pr - rq - pq}$ $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{\pi}{4}$ $\alpha + \beta + \gamma = \tan^{-1} \left(\frac{p + q + r - pqr}{1 - pr - rq - pq} \right), = \tan^{-1} \left(\frac{\frac{1}{3} + \frac{1}{4} + \frac{2}{9} - \frac{1}{3} \times \frac{1}{4} \times \frac{2}{9}}{1 - \frac{1}{3} \times \frac{1}{4} - \frac{1}{3} \times \frac{2}{9} - \frac{2}{9} \times \frac{1}{4}} \right) = \tan^{-1} \frac{85}{108}$ $= \tan^{-1} 1 = \frac{\pi}{4}$	
13 a)	$y = 72x + 3x^2 - 2x^3$ $\frac{dy}{dx} = 72 + 6x - 6x^2$, for max $\frac{dy}{dx} = 0$ So, $72 + 6x - 6x^2 = 0$, thus $x^2 - x - 12 = 0$ $(x - 4)(x + 3) = 0$ so $x = 4$, $x \neq -3$ For $x = 4$, $y = 288 + 48 - 128 = 208$	
b)	For points of integration to get the limits of integration, $12 = x(8 - x)$, $x^2 - 8x + 12 = 0$ $(x - 6)(x - 2) = 0$ so $x = 6$, $x = 2$, points are (2, 12), & (6, 12)	



$$\begin{aligned}
 A &= \int_2^6 (8x - x^2 - 12) dx \\
 &= \left[4x^2 - \frac{x^3}{3} - 12x \right]_2^6 \\
 &= (144 - 72 - 72) - \left(16 - \frac{8}{3} - 24 \right) = \frac{32}{3} \text{ sq. units}
 \end{aligned}$$

14 Turning points, $\frac{dy}{dx} = \frac{(x^2 - 3x + 2)(8x - 10) - (4x^2 - 10x + 7)(2x - 3)}{(x^2 - 3x + 2)^2}$

For $\frac{dy}{dx} = 0$, we have,

$$8x^3 - 10x^2 - 24x^2 + 30x + 16x - 20 - 8x^3 + 12x^2 + 20x^2 - 30x - 14x + 21 = 0$$

To get, $-2x^2 + 2x + 1 = 0$

$$x = \frac{-2 \pm \sqrt{2^2 - (-4 \times 2 \times 1)}}{-4} = \frac{1}{2} \pm \frac{\sqrt{3}}{2} \quad \text{so } x = -0.366, x = 1.366$$

Thus, $(1.366, -3.464)$ max, $(-0.366, 3.464)$ min

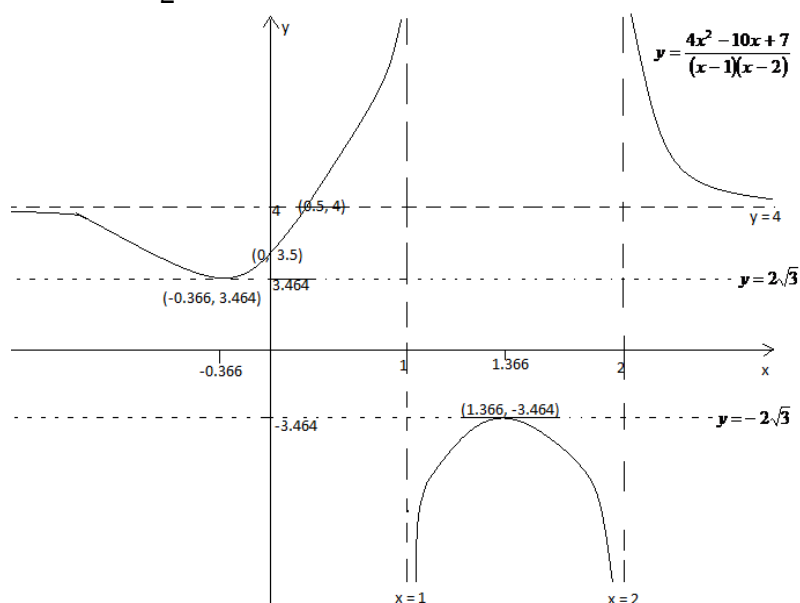
Intercepts: $x = 0, y = \frac{7}{2}$ so $(0, 3.5)$

$y = 0, 4x^2 - 10x + 7 = 0$ has no real roots since $(-10)^2 - (16 \times 7) < 0$

Vertical asymptotes: $x = 1, x = 2$

Horizontal asymptote, $y = 4, 4x^2 - 12x + 8 = 4x^2 - 10x + 7,$

$x = \frac{1}{2}$, thus curve crosses horizontal asymptote at $(0.5, 4)$



15 a)	$\mathbf{AP} : \mathbf{PB} = -5 : 3 \text{ or } 5 : -3$ $\Rightarrow +3 \mathbf{AP} = -5 \mathbf{PB}, \quad 3(\mathbf{AO} + \mathbf{OP}) = -5(\mathbf{PO} + \mathbf{OB})$ $3\mathbf{OP} - 5\mathbf{OP} = -5\mathbf{OB} + 3\mathbf{OA}$ $-2\mathbf{OP} = -5 \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix} + 3 \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}, \quad \mathbf{OP} = -\frac{1}{2} \left[\begin{pmatrix} 15 \\ -10 \\ -40 \end{pmatrix} + \begin{pmatrix} 6 \\ 12 \\ 18 \end{pmatrix} \right]$ $\mathbf{OP} = \begin{pmatrix} -21/2 \\ -1 \\ 11 \end{pmatrix} \text{ so } \mathbf{OP} = \frac{-21}{2} \mathbf{i} - \mathbf{j} + 11\mathbf{k}.$	
b)	Let $\mathbf{u} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 7 \\ -6 \end{pmatrix}$ and let $\mathbf{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ be the normal to $\mathbf{u}$ & $\mathbf{v}$. $\Rightarrow x - y - 2z = 0 \dots \text{(i)} \text{ and } 3x + 7y - 6z = 0 \dots \text{(ii)}$ Using (i) and (ii) $7x - 7y - 14z = 0, \quad 3x + 7y - 6z = 0, \text{ to get } x = 2z, \text{ thus from (i)}$ $y = 0$ So $\mathbf{a} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = 2z\mathbf{i} + 0\mathbf{j} + z\mathbf{k} = z(2\mathbf{i} + \mathbf{k})$ $\Rightarrow \text{normal vector is } 2\mathbf{i} + \mathbf{k}$ But $ax + by + cz = ax_o + by_o + cz_o$ $\Rightarrow 2x + 0 + z = 0 + 0 + 0$ $\therefore 2x + z = 0.$	
16 .	$\frac{x^2}{a^2} + \frac{(mx + c)^2}{b^2} = 1$ $(a^2m^2 + b^2)x^2 + 2a^2cmx + a^2c^2 - a^2b^2 = 0$ For a tangent, $B^2 - 4AC = 0$, so $4a^4c^2m^2 - 4a^2(a^2m^2 + b^2)(c^2 - b^2) = 0 \text{ for } 4a^2b^2c^2 \neq 0, \text{ then}$ $a^2c^2m^2 - a^2c^2m^2 + a^2m^2b^2 - b^2c^2 + b^4 = 0$ $c^2 = a^2m^2 + b^2$ For $\frac{x^2}{16} + \frac{y^2}{9} = 1$, $a^2 = 16$, $b^2 = 9$ $c^2 = 16m^2 + 9, \text{ and for } (-3, 3), \quad -3 = 3m + c, \text{ so, } c = -3 - 3m$ $16m^2 + 9 = 9 + 18m + 9m^2, \quad m(7m - 18) = 0 \text{ thus}$ $m = 0, \quad \& \quad m = \frac{18}{7}, \text{ similarly, } c = -3, \quad \& \quad c = -\frac{75}{7}$ Equations are $y = -3$ and $7y = 18x - 75$	